REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*
04/12/07

2. REPORT TYPE
Final Technical Report

3. DATES COVERED *(From - To)*
2004-2007

4. TITLE AND SUBTITLE
A Study of Uncertainties in Nonlinear Aeroelastic Systems

5a. CONTRACT NUMBER
FA9550-04-1-0071

5b. GRANT NUMBER
313-6041

5c. PROGRAM ELEMENT NUMBER
NA

6. AUTHOR(S)
Earl H. Dowell

5d. PROJECT NUMBER
NA

5e. TASK NUMBER
NA

5f. WORK UNIT NUMBER
NA

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Duke University
Mech. Eng. and Materials Sciences
Pratt School of Engineering
P.O. Box 90300
Durham, NC 27708

8. PERFORMING ORGANIZATION REPORT NUMBER
NA

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Air Force Office of Scientific Research
875 N. Randolph St. Suite 325, Room 3112
Arlington, VA 22203-1768

10. SPONSOR/MONITOR'S ACRONYM(S)
AFOSR

AFRL-SR-AR-TR-07-0210

12. DISTRIBUTION / AVAILABILITY STATEMENT

No Limitations
Approved for Public Release - Distribution A

13. SUPPLEMENTARY NOTES
NA

14. ABSTRACT

A principal goal of the Air Force research program is to develop accurate, reliable and efficient computational models so that laboratory and flight tests may be decreased and/or the results of such tests may be more effective in reducing design and production costs of new Air Force systems. Key challenges in achieving this goal are the uncertainties that exist in computational models. These uncertainties may be a result of the limitations of the mathematical model in representing physical reality, the numerical uncertainties that arise in obtaining (computational) solutions to these models, and/or the uncertainties in the parameters that enter into such models.

Examples of each of these uncertainties have been studied for nonlinear aeroelastic systems and the computational results compared with experiment. New highly computationally efficient methods have been developed.

15. SUBJECT TERMS
Aeroelasticity, Dynamics, Nonlinearities, Uncertainties

16. SECURITY CLASSIFICATION OF: NA			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
					Earl H. Dowell
a. REPORT	b. ABSTRACT	c. THIS PAGE	SAR	24	19b. TELEPHONE NUMBER *(include area code)*
NA	NA	NA			919-660-5302

A STUDY OF UNCERTAINTIES IN NONLINEAR AEROELASTIC SYSTEMS

AFOSR GRANT NUMBER FA9550-04-1-0071

PROGRAM MANAGER: VICTOR GIURGIUTIU
PROGRAM INVESTIGATORS FOR DUKE UNIVERSITY:
EARL DOWELL
PETER ATTAR*
JEFFREY THOMAS

SEPTEMBER, 2006

* Dr. Attar was a PhD student at Duke University, then post-doctoral fellow at the Air Force Research Laboratory and is now an Assistant Professor, University of Oklahoma

ABSTRACT

MOTIVATION

A principal goal of the Air Force research program is to develop accurate, reliable and efficient computational models so that laboratory and flight tests may be decreased and/or the results of such tests may be more effective in reducing design and production costs of new Air Force systems. Key challenges in achieving this goal are the uncertainties that exist in computational models. These uncertainties may be a result of the limitations of the mathematical model in representing physical reality, the numerical uncertainties that arise in obtaining (computational) solutions to these models, and/or the uncertainties in the parameters that enter into such models.

Examples of each of these uncertainties include the following.

PHYSICAL UNCERTAINTIES:

At the Reynolds numbers of interest for aerospace vehicles, there is usually an empirical turbulence model introduced in order to obtain solutions to the Navier-Stokes equations and this leads to uncertainties in the results. Also in structural models, approximations are made based upon the anticipated magnitude of the structural deformation that lead progressively from small deflection (linear) models to small strain (but still elastic nonlinear) models to large strain (plastic) models. This also leads to uncertainty in the results.

NUMERICAL UNCERTAINTIES:

Whether one uses modal methods or finite element (or volume) methods to solve the partial differential equations for the fluid or structure, the number of modes or elements

required to give a given level of accuracy requires a convergence study. Unless such a study is done there remains some numerical uncertainty in the answer and a priori there is no way of determining the accuracy of the answer in the absence of a convergence study.

PARAMETER UNCERTAINTIES:

Because of manufacturing and fabrication tolerances there is always some uncertainty in the parameters of a model, e.g. there may be uncertainties in the wing shape that will impact the aerodynamic flow and there may be uncertainties in the dimensions of the structure that may impact its stiffness, mass and/or damping characteristics.

RESEARCH PLAN:

Representative examples of all of these uncertainties have been addressed in the present work. For example, we have systematically improved the structural model and determined its impact on the prediction of limit cycle oscillations of wings which have been experimentally investigated in previous work by the present authors and others.

We have also studied the numerical convergence of solutions using both modal and finite element methods.

And finally we have studied the impact of uncertainties of parameters such as thickness and modulus of elasticity.

Because all such calculations are computationally intensive, special attention has been given to developing new effective solution techniques that provide the required numerical accuracy in the solution and allow a range of parameter uncertainty to be investigated using the highest (as well as lower) fidelity computational models.

REPRESENTATIVE RESULTS:

A fuller account of our results is given in the publications in the list of references cited at the end of this abstract. Here a few representative highlights are provided to summarize some of the key findings of our work.

AN EXAMPLE OF A PHYSICAL UNCERTAINTY

A comparison between theory and experiment is shown for a cropped delta wing undergoing limit cycle oscillations (LCO). Figure 1 shows the wing geometry and Figure 2 is a plot of wing deformation versus flow dynamic pressure. The experiment was performed at NASA Ames Research Center and earlier theoretical/computational studies were done by Ray Gordnier and his colleagues at AFRL. In the first such studies a nonlinear fluid model (Euler or Navier-Stokes equations) was used in conjunction with a small deflection, linear structural model. The result was that the theoretical model gave LCO response levels more than an order of magnitude larger than experiment. Clearly there was something missing in the theoretical model (a physical uncertainty).

At the suggestion of the PI, a nonlinear structural model was investigated (the Von Karman plate theory). This led to substantial improvement in the correlation between theory and experiment and it was definitely established that for this experimental model the structural nonlinearity was dominant relative to the fluid nonlinearity. However some differences between computational and test results remained. The Von Karman plate theory is a small rotation theory and so the next step has been to used a finite element model that allows for larger rotations. The ANSYS finite element code has been used for this purpose. The results from experiment, the Von Karman theoretical models and ANSYS are all shown in Figure 1. The result is a further improvement in the agreement between theory and experiment, though some differences remain. Thus further study is indicated, although it is well to remember that there are uncertainties in experiments as well as computational models. One of the advantages of a computational model is that it may be refined and redone. While that is also possible in principle for an experiment, in practice it is usually far more difficult to repeat an experiment such as a wind tunnel test or a flight test. See Reference 3 for more details.

AN EXAMPLE OF A NUMERICAL UNCERTAINTY

In related work on another delta wing model that was tested in the Duke University Wind Tunnel, the Von Karman plate model was used but the solution was obtained with a modal analysis rather than a finite element analysis. See Figure 3 for the wing geometry. The finding was that a surprisingly large number of modes were needed to obtained sufficient numerical accuracy. In Figure 4 representative results are shown from experiment, from the ANSYS finite element computational code, and from a modal solution of the Von Karman plate model. This is a plot of wing deformation versus flow velocity. While there is good agreement for the flow velocity at which LCO begins, there are substantial differences with respect to LCO response (wing deformation) per se. The differences between the two computational models in this case reflect both the physical differences in the underlying theoretical models and also numerical differences due to the different solution methods. See Reference 4 for more details.

AN EXAMPLE OF PARAMETER UNCERTAINTY

When a parameter uncertainty exists it is the usual practice to assume the uncertainty can be characterized as a random distribution of the parameter about a mean or nominal value. The challenge then is to find the corresponding random distribution of the system response. While in principle this can be done by repeatedly simulating the response for each random choice of the parameter (a Monte Carlo simulation), in practice such computations are often impractical for the complex systems of interest to the Air Force. Thus other methods are being developed including response surface techniques that seek to develop a relatively simple and computationally compact relationship between the uncertain parameter and the system response. In the present work an efficient method for constructing response surfaces has been developed and a representative result is shown in Figure 5. Here is a plot of the probability of occurrence versus the level of LCO response for a given flow velocity for the Duke delta wing model. Also shown on the plot is the

nominal or mean response if there were no uncertainty in the parameter which in this case is the thickness of the delta wing model. Results are shown from both a brute force Monte Carlo simulation as well as from the new response surface method. Such a plot gives the analyst or designer a measure of the uncertainty in system response due to the uncertainty in the parameter.

Equally important, the new response surface computational method developed for obtaining such results is several orders of magnitude faster than a Monte Carlo simulation while providing adequate accuracy. See References 5 and 6 for more details.

ANOTHER EXAMPLE OF PARAMETER UNCERTAINTY AND PHYSICAL MODELING UNCERTAINTY:

In very recent work on F-16 linear and nonlinear aeroelastic response, the sensitivity to

- aerodynamic modeling of tip missiles (physical modeling uncertainty,

and

- small changes in structural frequencies (parameter uncertainty)

has been shown. See Reference 7. For example, in Figure 6 the computational results for a nominal set of structural frequencies and those for a 1% change in such frequencies are shown compared to flight test data. Clearly small uncertainties in structural frequencies can lead to significant uncertainties in system response. A 1% change in a key structural frequency can lead to a 10% change in the Mach number at which flutter or limit cycle oscillations occur, for example.

SUMMARY:

The effects of uncertainties in the physical/computational model, the numerical solution, and the parameters that are inputs to the model have all been studied. Representative results have been shown including correlations with experiment and their significance discussed. New methods have been developed for determining solutions to high fidelity physical/computational models, determining their numerical accuracy and taking into account parameter uncertainties. These new methods are much faster than previous methods while maintaining the levels of accuracy required.

PUBLICATIONS:

1. Dowell, E.H., Edwards, J.W., and Strganac, T.W., "Nonlinear Aeroelasticity," *Journal of Aircraft*, Vol. 40, pp. 857-874, Sept-Oct 2003.

2. Attar, P.J., Dowell, E.H., and Tang, D.M., "A Theoretical and Experimental Investigation of the Effects of a Steady Angle of Attack on the Nonlinear Flutter of a Delta Wing Plate Model," *Journal of Fluids and Structures*, Vol. 17, pp. 243-259, February, 2003.

3. Attar, P.J. and Gordnier, R.E., "Aeroelastic Prediction of the Limit Cycle Oscillations of a Cropped Delta Wing," *Journal of Fluids and Structures*, Vol. 22, pp. 45-58, January 2006.

4. Attar, P.J., Dowell, E.H., and Tang, D.M., "Modeling Delta Wing Limit Cycle Oscillations Using a High Fidelity Structural Model," *Journal of Aircraft*, Vol. 42, pp. 1209-1217, September-October 2005.

5. Attar, P.J., and Dowell, E.H., "A Stochastic Analysis of the Limit Cycle Behavior of a Nonlinear Aeroelastic Model Using the Response Surface Method," Presented at the 46th AIAA SDM Conference, Austin, TX, April 2005.

6. Attar, P.J. and Dowell, E.H., "A Reduced Order System ID Approach to the Modeling of Nonlinear Structural Behavior in Aeroelasticity," *Journal of Fluids and Structures*, Vol. 21, pp. 531-542, December 2005.

7. Thomas, J.P., Dowell, E.H., Hall, K.C., and Denegri, Jr., C.M., *Virtual Aeroelastic Flight Testing for the F-16 Fighter with Stores*, AIAA Paper 2007-1640, AIAA/U.S. Air Force T&E Days, Destin, FL, February 2007.

EXPERIMENTAL CROPPED DELTA WING MODEL

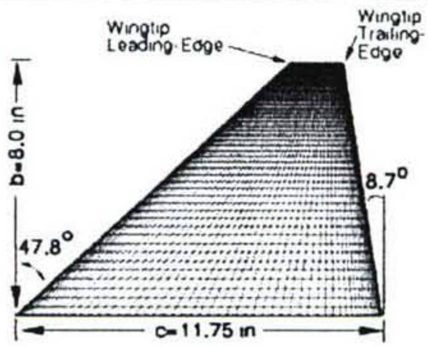


FIGURE 1

UNCERTAINTIES IN PHYSICAL MODELING (HIGHER ORDER STRUCTURAL THEORY IMPROVES AGREEMENT WITH EXPERIMENT)

Wingtip Trailing Edge LCO Amplitude vs. Dynamic Pressure

FIGURE 2

Experimental Delta Wing Model in Duke University Wind Tunnel

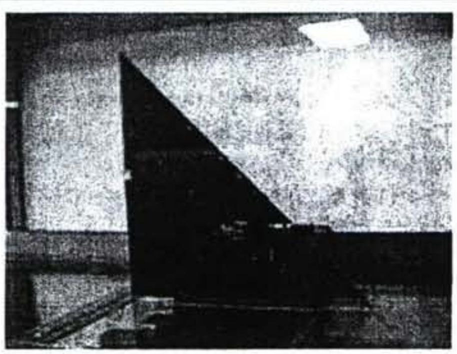
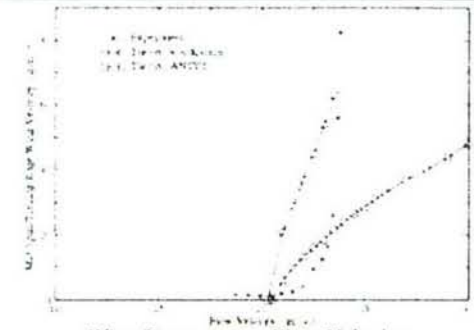


FIGURE 3

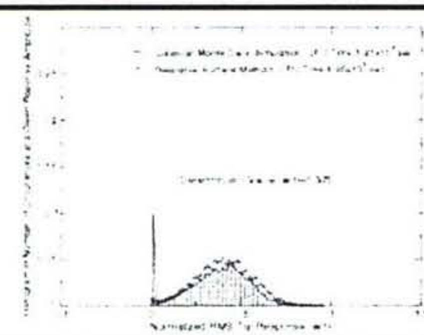
UNCERTAINTIES IN PHYSICAL MODELING AND NUMERICAL MODELING (MODAL ANALYSIS WITH VON KARMAN STRUCTURAL THEORY CONVERGES SLOWLY)



Wing Response vs. Flow Velocity

FIGURE 4

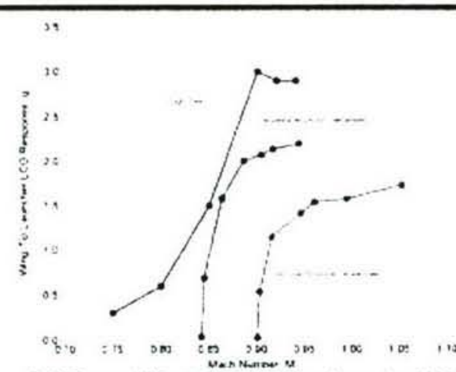
UNCERTAINTIES IN PHYSICAL PARAMETERS



Probability versus RMS Amplitude for a Flow Velocity of 29 m/s

FIGURE 5

LCO Response vs Mach Number: Sensitivity to Uncertainty in Structural Natural Frequencies



F-16 Forward Wingtip Launcher Accelerometer (2000 feet)

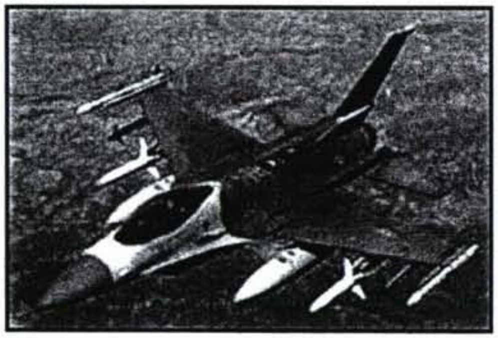
FIGURE 6

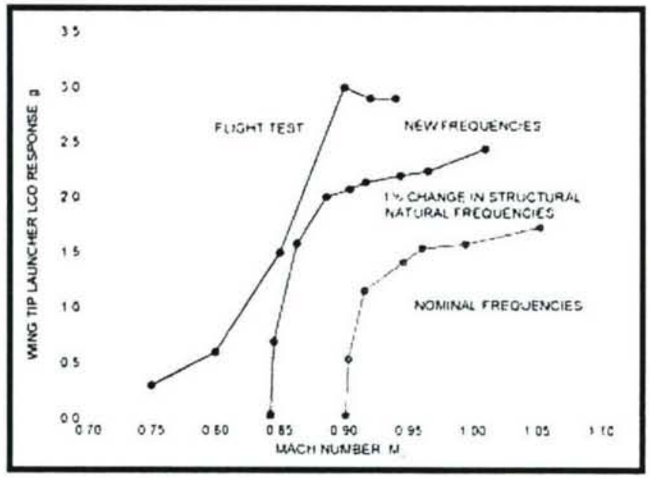
A STUDY OF UNCERTAINTIES IN NONLINEAR AEROELASTIC SYSTEMS

- Goal: Develop highly efficient computational methods to assess the sensitivity of aeroelastic response to uncertainties.

- UNCERTAINTIES IN
  - PHYSICAL MODELING
    - FLUID TURBULENCE
    - STRUCTURAL DAMPING
    - STRUCTURAL FREQUENCIES
    - NONLINEARITIES
  - COMPUTATIONAL METHODS
    - GRID RESOLUTION
    - MODE CONVERGENCE
  - SYSTEM PARAMETERS
    - MATERIAL CONSTANTS
    - GEOMETRY
    - COMPONENT INTERCONNECTIONS (JOINTS)

KEY QUESTION: HOW SENSITIVE IS THE AEROELASTIC SYSTEM RESPONSE TO EACH UNCERTAINTY AND, THUS, WHICH ARE MOST IMPORTANT?





SENSITIVITY OF F-16 LIMIT CYCLE OSCILLATIONS (LCO) TO UNCERTAINTIES IN STRUCTURAL FREQUENCIES: 1% CHANGE IN STRUCTURAL FREQUENCY PRODUCES SIGNIFICANT CHANGE IN LCO RESPONSE

- Payoff to the Air Force
- Systematic assessment of effect of system uncertainties on aeroelastic response
- Dramatic reduction of computation time and cost for nonlinear aeroelastic analysis and design

PI: Earl Dowell

Mechanics of Materials & Devices and Structural Mechanics Program Review

A STUDY OF UNCERTAINTIES IN NONLINEAR AEROELASTIC SYSTEMS

AFOSR Grant Number FA9550-04-1-0071

PROGRAM MANAGER: VICTOR GIURGIUTIU

PROGRAM INVESTIGATORS:

EARL DOWELL

PETER ATTAR*

JEFFREY THOMAS

DUKE UNIVERSITY

SEPTEMBER, 2006

*Dr. Attar was a post-doctoral fellow at the Air Force Research Laboratory (AFRL) and is now an Assistant Professor at the University of Oklahoma

OUTLINE OF TALK

OBJECTIVES

CURRENT PRACTICE AND LIMITATIONS

WHAT IS NEW IN OUR APPROACH?

WHAT IS THE IMPACT?

WHAT ARE THE RISKS AND REWARDS?

PROGRESS TO DATE AND NEXT STEPS

OBJECTIVES

DEVELOP *A RAPID SOLUTION METHOD WITH STATE OF THE ART* PHYSICAL MODELING AND NUMERICAL *ACCURACY* FOR SENSITIVITY ANALYSIS TO DETERMINE THE UNCERTAINTIES IN AEROELASTIC REPONSE DUE TO *UNCERTAINTIES* IN

- *PHYSICAL MODELING*
- *COMPUTATIONAL METHODS*
- *SYSTEM PARAMETERS*

CURRENT PRACTICE AND LIMITATIONS

CURRENT AEROELASTIC COMPUTATIONAL METHODS ARE *TOO SLOW* TO CARRY OUT SENSITIVITY ANALYSIS OF UNCERTAINTIES

WHAT IS NEW IN OUR APPROACH?

- A *SYSTEM IDENTIFICATION METHOD* IS USED IN CONJUNCTION WITH HIGH FIDELITY MODELS OF THE FLUID AND STRUCTURE TO DEVELOP GLOBAL RELATIONSHIPS BETWEEN AEROELASTIC SYSTEM INPUTS AND OUTPUTS *(RESPONSE SURFACE)*

- A NEW *RAPID SOLUTION METHOD* FOR COMPUTATIONAL FLUID DYNAMICS *(CFD)* IS USED TO *DETERMINE AEROLASTIC REPONSE SURFACE RELATIONSHIP*

WHAT IS THE IMPACT?

- CURRENT AND FUTURE AIR FORCE FLIGHT VEHICLES HAVE AEROELASTIC RESPONSES THAT ARE SENSITIVE TO UNCERTAINTIES. MANUFACTURING AND MAINTENANCE QUALITY CONTROL REQUIREMENTS ARE SIGNIFICANTLY IMPACTED BY THESE UNCERTAINTIES AS ARE FLIGHT SAFETY MARGINS

WHAT ARE THE RISKS AND REWARDS?

THE REWARDS ARE THAT *NONLINEAR AEROELASTIC COMPUTATIONS ARE ORDERS OF MAGNITUDE FASTER*, THEREBY MAKING POSSIBLE SENSITIVITY ANALYSIS USING *STATE OF THE ART HIGH FIDELITY MODELS.*

THE RISKS ARE ??????

PROGRESS TO DATE

- REPONSE SURFACES HAVE BEEN CONSTRUCTED FOR A DELTA WING AEROELASTIC MODEL THAT ALLOW RAPID DETERMINATION OF RANDOM DISTRIBUTION OF RESPONSE TO RANDOM PARAMETER VARIATIONS

- RESULTS HAVE BEEN OBTAINED FOR *THREE DIFFERENT F-16 WING/STORE CONFIGURATIONS* FOR THE FLUTTER BOUNDARY AND LIMIT CYCLE OSCILLATIONS USING A STATE OF THE ART RANS/CFD CODE AND THE RESULTS COMPARED TO FLIGHT TEST DATA. OUR APPROACH MAKES SUCH CALCULATIONS FEASIBLE AND THEY ARE THE ONLY SUCH RESULTS OBTAINED TO DATE BY ANY COMPUTATIONAL METHOD.

THE AGREEMENT BETWEEN THEORETICAL COMPUATIONS AND FLIGHT TEST MEASUREMENTS IS ENCOURAGING

- SENSITIVITY ANALYSIS FOR THE F-16 HAS BEEN PERFORMED FOR UNCERTAINTIES IN

  -PHYSICAL MODELING (AERODYNAMICS OF TIP MISSILES, EULER VS NAVIER STOKES EQ)
  -SYSTEM PARAMETERS (ANGLE OF ATTACK, STRUCTURAL NATURAL FREQUENCIES)

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH · UNITED STATES AIR FORCE

NEXT STEPS

DUKE
EDMUND T. PRATT, JR. SCHOOL OF ENGINEERING

- COMBINE RESPONSE SURFACE SYSTEM IDENTIFICATION ALGORITHM WITH STATE OF THE ART CFD-BASED AEROELASTIC MODELS

- DEMONSTRATE NEW CAPABILITY ON CURRENT (F-16) AND FUTURE (F-22, F-35) AIRCRAFT

UNCERTAINTIES IN NONLINEAR AEROELASTIC SYSTEMS

UNCERTAINTIES IN THE PHYSICS OF AEROELASTIC MODELING

- FLUID TURBULENCE MODELS
- STRUCTURAL DAMPING
- NONLINEARITIES IN THE FLUID AND/OR STRUCTURE

UNCERTAINTIES IN COMPUTATIONAL AEROELASTIC MODELS

- CONVERGENCE OF REDUCED ORDER MODELS
- COMPUTATIONAL GRID RESOLUTION
- NONLINEARITES AND THEIR EFFECTS ON GRID AND MODEL (MODAL) CONVERGENCE

UNCERTAINTIES IN PARAMETERS

- MATERIAL CONSTANTS
- GEOMETRICAL DIMENSIONS
- JOINT STIFFNESSES

SENSITIVITY ANALYSIS OF AEROELASTIC SYSTEMS

KEY QUESTION: HOW SENSITIVE IS THE AEROELASTIC RESPONSE TO UNCERTAINTIES IN THE AEROELASTIC MODEL?

FOR EXAMPLE, IT IS NOW KNOWN THAT THE F-16 FLUTTER AND LIMIT CYCLE OSCILLATION (LCO) RESPONSE IS MORE SENSITIVE TO SMALL UNCERTAINTIES IN KEY STRUCTURAL MODAL FREQUENCIES THAN TO LARGER UNCERTAINTIES IN STRUCTURAL DAMPING.

AS ANOTHER EXAMPLE, IT IS ESSENTIAL TO INCLUDE AN ACCURATE REPRESENTATION OF NONLINEARITIES TO MODEL LCO RESPONSE. AND IN GENERAL FLUID NONLINEARTIES ARE MORE SENSITIVE TO PARAMETER UNCERTAINTIES THAN ARE STRUCTURAL NONLINEARITIES

KEY CHALLENGE: DEVELOP HIGHLY COMPUTATIONALLY EFFICIENT METHODS TO DO SENSITIVITY ANALYSIS. THE CURRENT GRANT IS DIRECTED TOWARD MEETING THIS CHALLENGE.

THE ULTIMATE ARBITER: CORRELATION OF THEORY AND EXPERIMIENT WIND TUNNEL TESTS ARE PART OF THE CURRENT GRANT ACTIVITY.

INNOVATIONS IN SCIENCE

- CONDUCTED TRANSONIC FLUTTER AND LCO STUDIES USING HIGH FIDELITY FLUID AND STRUCTURAL COMPUTATIONAL MODELS TAKING INTO ACCOUNT PHYSICAL, NUMERICAL AND PARAMETER UNCERTAINITES
- CORRELATED RESULTS FROM COMPUTATION AND EXPERIMENT
- DEVELOPED NEW METHODS TO INCREASE COMPUTATIONAL PRODUCTIVITY BY SEVERAL ORDERS OF MAGNITUDE (ESPECIALLY IMPORTANT FOR STUDIES OF UNCERTAINTY)

INNOVATION IN DESIGN

DEVELOPED COST EFFECTIVE COMPUTATIONAL MODELS THAT INCLUDE THE EFFECTS OF PHYSICAL, NUMERICAL AND PARAMETER UNCERTAINTIES

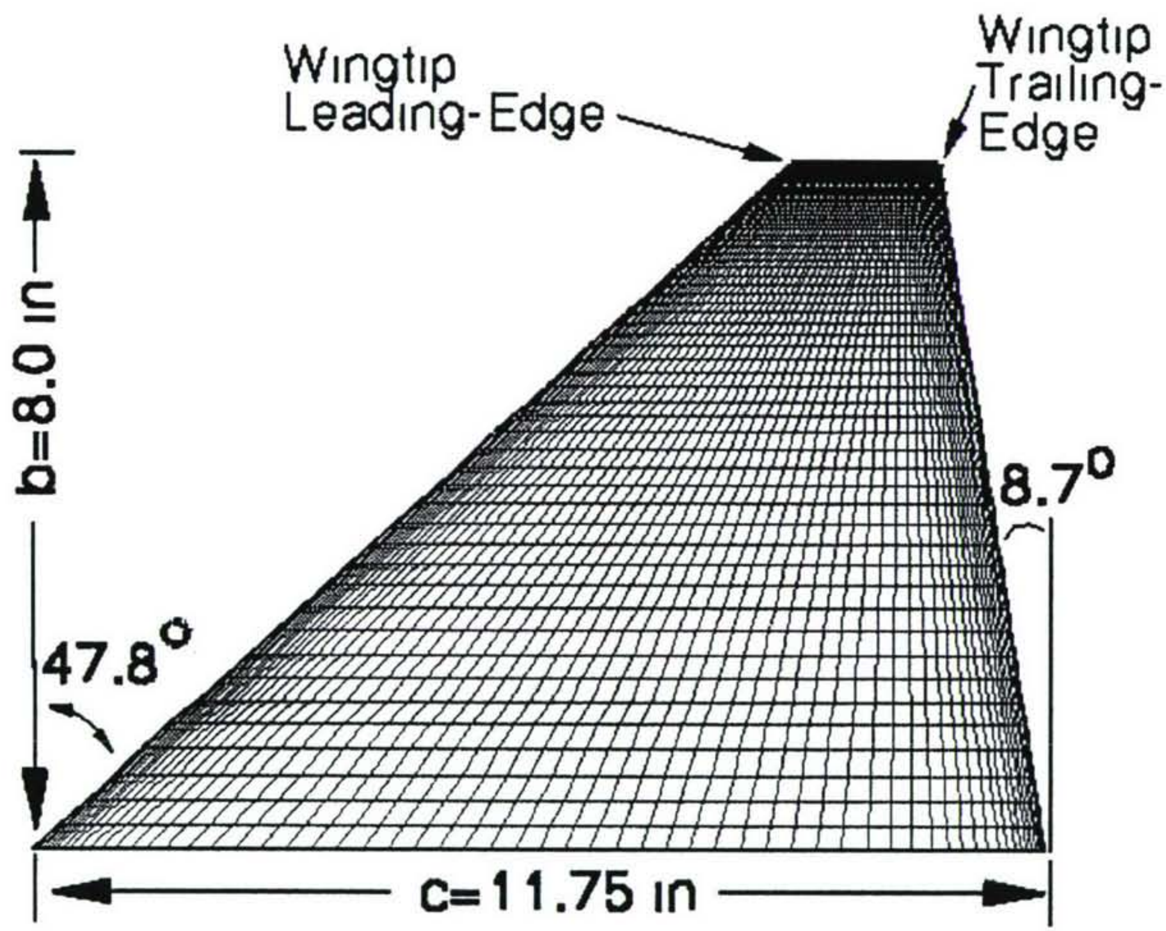
RELATED WORK AND PARTNERSHIPS

- AFRL has conducted significant work on parameter uncertainty and the present work complements that effort and advances the state-of-the-art in developing computational methods for higher fidelity models in the presence of physical, numerical and parameter uncertainties.

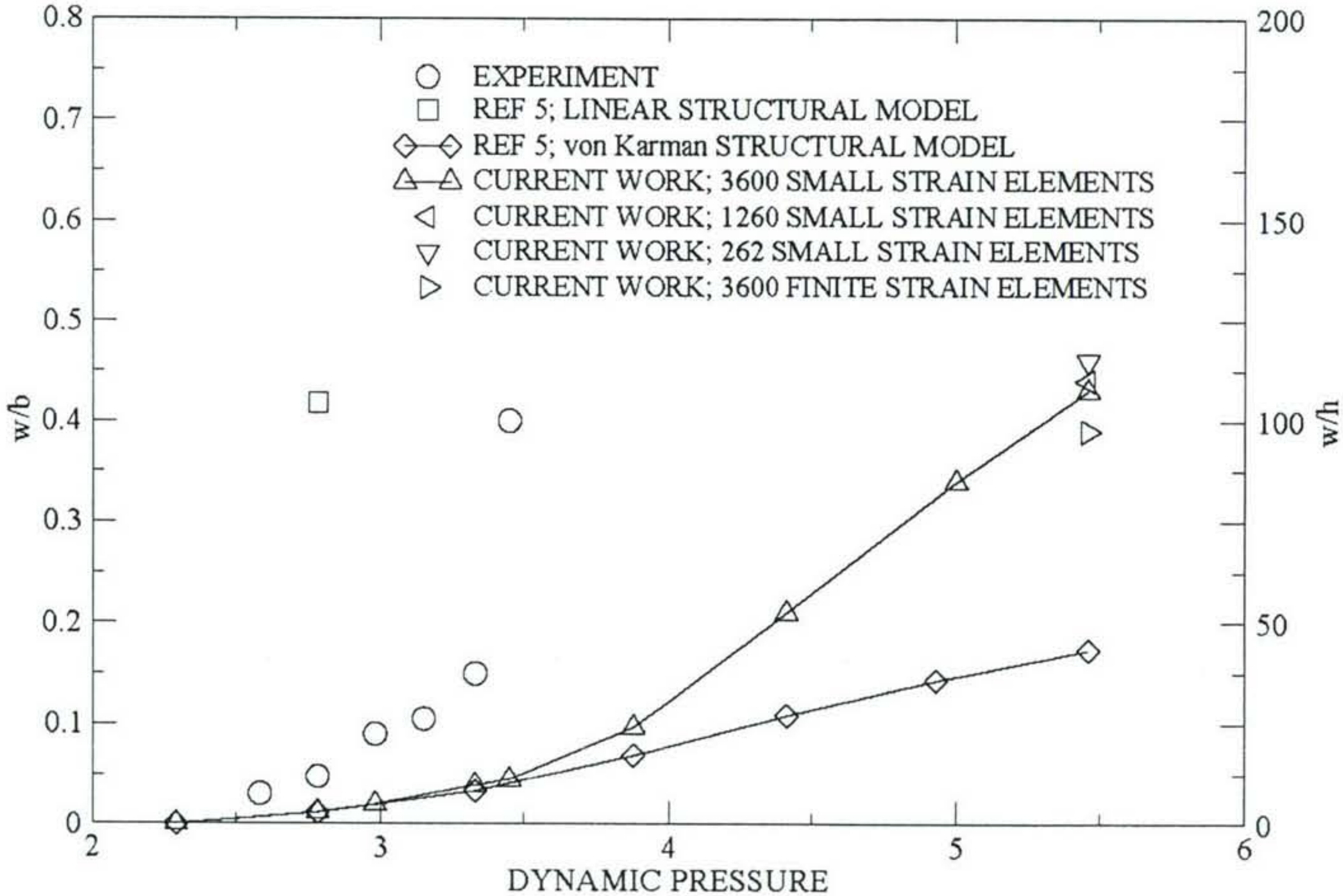
MOTIVATION AND RELEVANCY

- Reluctance to rely on computational methods for aircraft design is partially due to uncertainty
- Three types of uncertainty in computations:
  - Uncertainty in Physics Modeling
    - Correct equations?
    - Are boundary conditions sufficient?
    - Are initial conditions correct?
  - Uncertainty in numerical modeling
    - Is solution converged in both time and space discretization?
  - Uncertainty in physical parameters (parametric uncertainty)

EXPERIMENTAL CROPPED DELTA WING MODEL

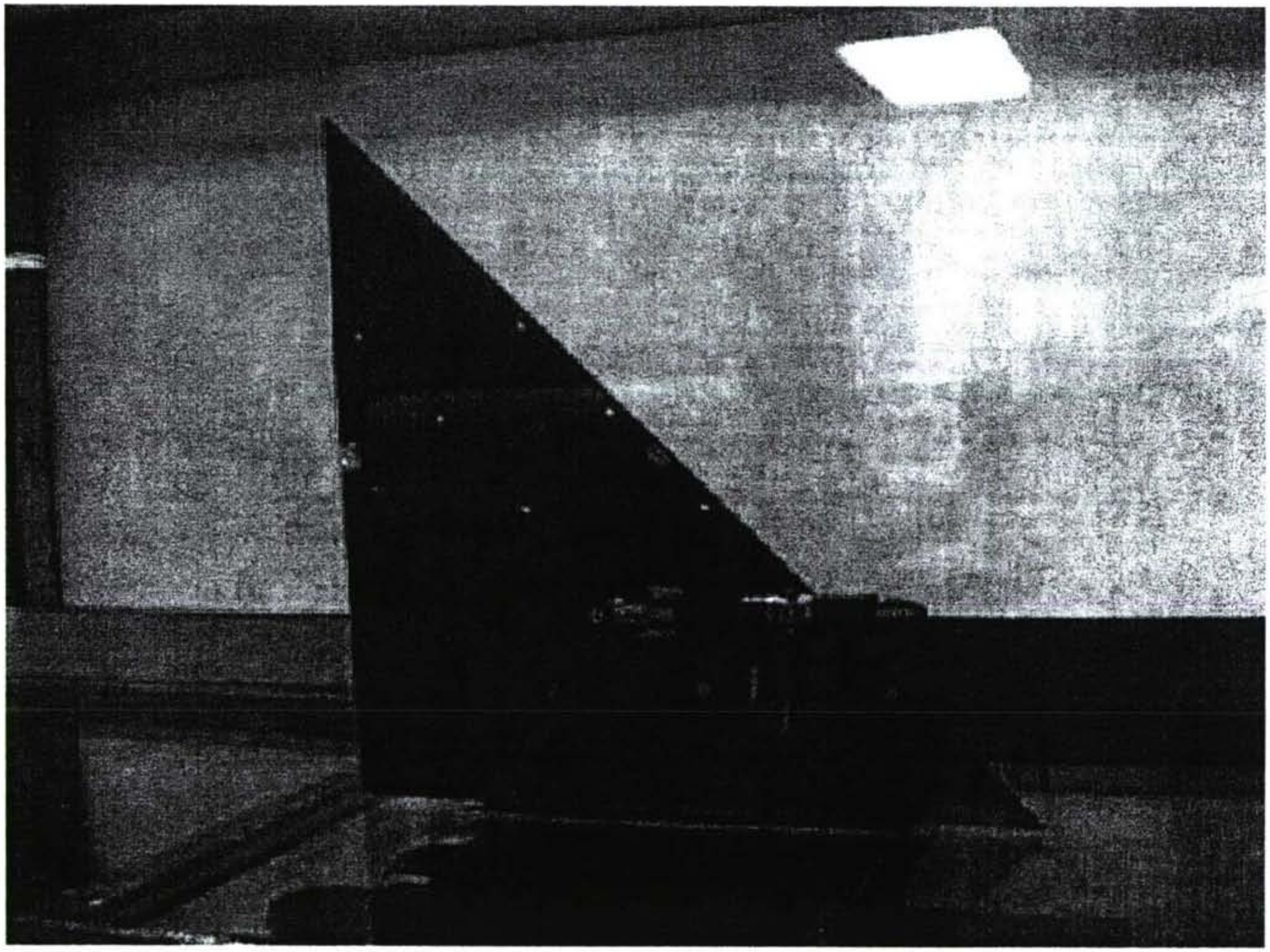


UNCERTAINTIES IN PHYSICAL MODELING (HIGHER ORDER STRUCTURAL THEORY IMPROVES AGREEMENT WITH EXPERIMENT)

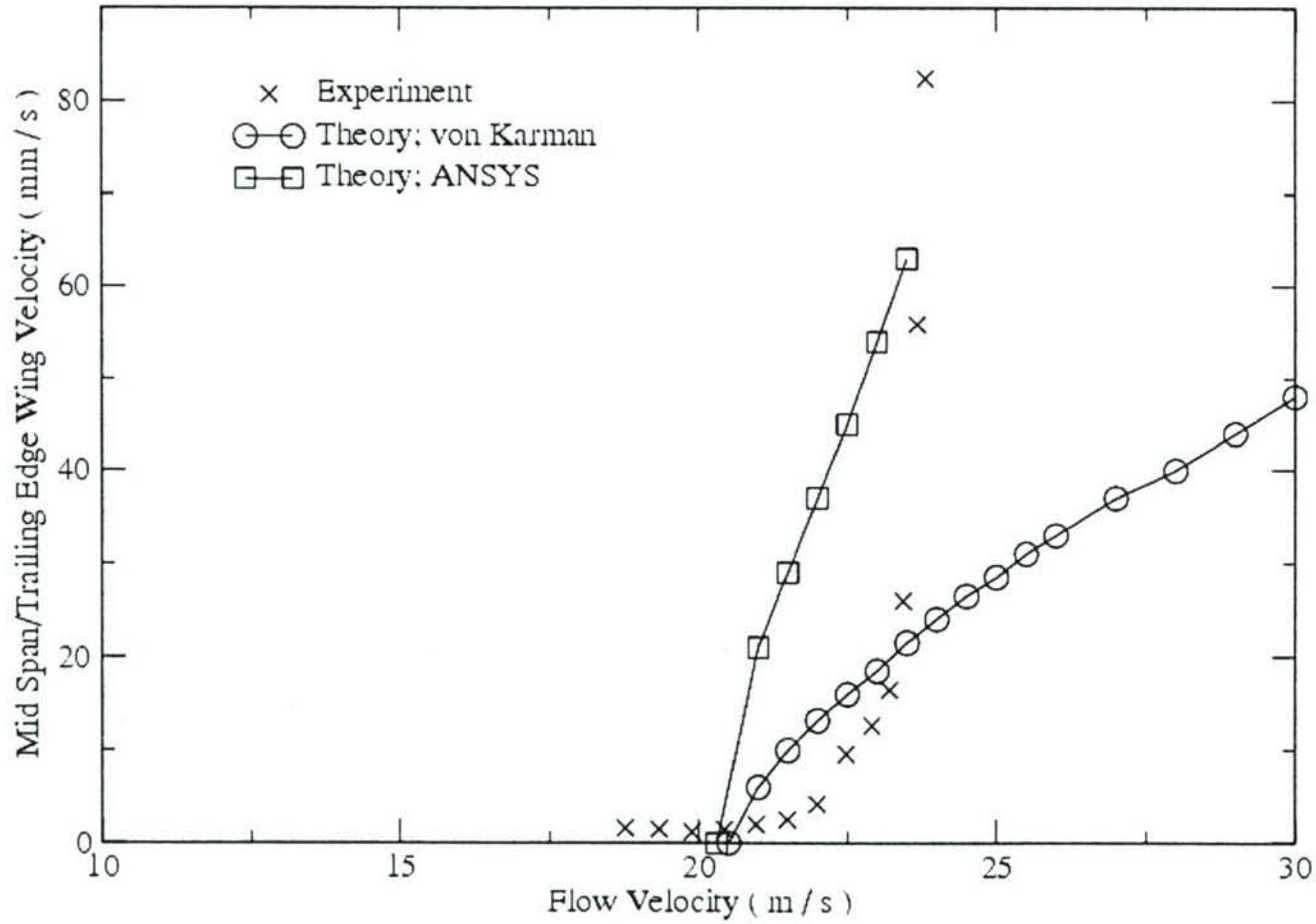


Wingtip Trailing Edge LCO Amplitude vs. Dynamic Pressure

Experimental Delta Wing Model in Duke University Wind Tunnel



UNCERTAINTIES IN PHYSICAL MODELING AND NUMERICAL MODELING (MODAL ANALYSIS WITH VON KARMAN STRUCTURAL THEORY CONVERGES SLOWLY)



Wing Response vs. Flow Velocity

Uncertainties in Physical Parameters

- **Uncertainty more problematic in nonlinear problems due to possible destabilizing effects**
- **Classical aeroelastic stochastic analysis use Monte Carlo simulation of governing equations**
  - Computationally expensive
- **Reduced order stochastic methods such as First and Second Order Reliability methods often used in stochastic design**
  - Often do not perform well if physics is highly nonlinear

Response Surface Methodology (RSM)

- **Basic idea: Represent system output, y, as a function of known input parameters, ξ_i :**

$$y = f(\xi_1, \xi_2, \ldots \xi_n) + \varepsilon$$

- **True form of *f* often unknown and very complicated**
  - Second order polynomial used here:

$$y = A_1 + \sum_{i=1}^{n} B_i \xi_i + \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} \xi_i \xi_j$$

RSM continued

- **In aeroelastic systems both subcritical and supercritical bifurcation must be accounted for.**
  - Bifurcation $\rightarrow$ discontinuity in the response surface function
- **Discontinuity modeled using a two region regression**
  - Coefficients A_1, B_i and C_{ij} computed for two separate input parameter regions
- **Monte Carlo simulation performed on polynomial model to gather statistical information**

Key Points of Current Methodology

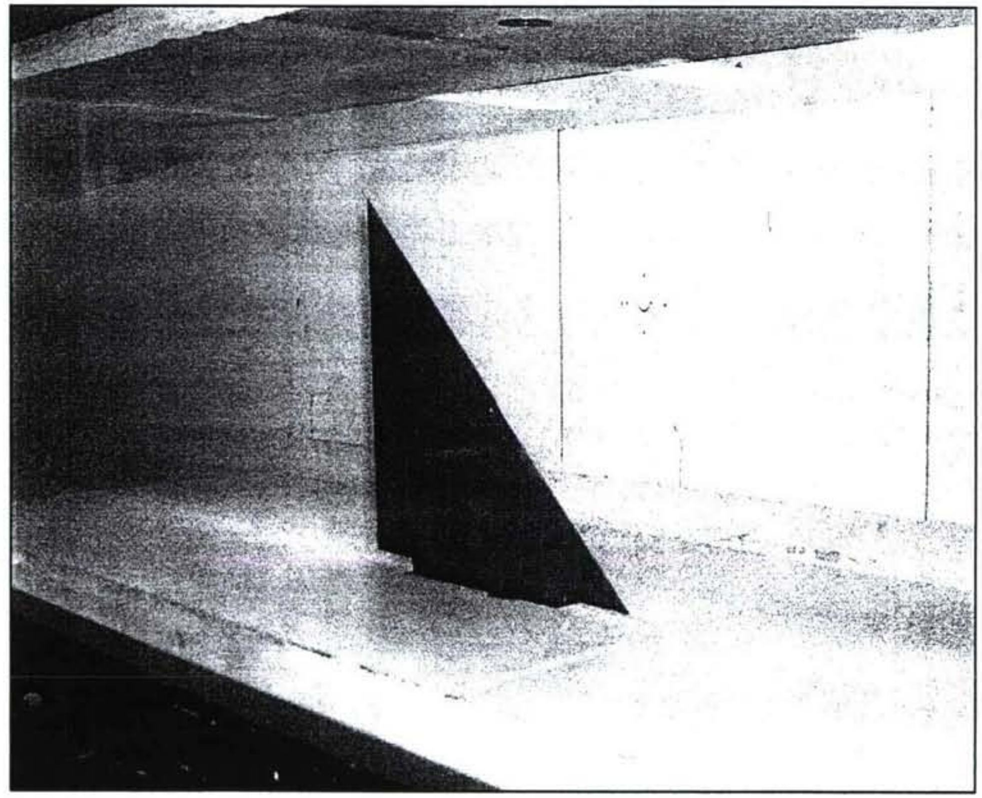
- **Region separation point/line computed "on the fly"**
  - Only viable for 1 or 2 random input parameters and a known switch value of system output y
  - More than 2 input parameters or unknown switch value "knots" must be chosen apriori
- **Constant coefficients A1, Bi and Cij computed using data from a *limited* number of Monte Carlo simulations of governing equations**
- **Method is non-intrusive**
  - Governing equations are not modified
- **Output quantity (y) can be any time-averaged or min/max quantity**

Aeroelastic Modeling

- **Structural model**
  - Modal solution of nonlinear von Karman plate equations
- **Aerodynamic model**
  - Flow field assumed to be a potential flow
  - Linear vortex lattice model used to solve flow model
  - Linear Bernoulli equation used to compute pressure difference on wing surface
- **Subiterations used to couple structural and fluid models**

Physical Model

- 45 degree flat plate delta wing
- Plexiglas material
- Middle 60% of root chord fully clamped



Computational Model

- **Uncertainty modeling:**
  - Two input parameters chosen to be random: plate thickness and material elastic modulus
  - Random input parameters assumed to have Gaussian probability distribution with zero mean and a standard deviation of 1
  - Effects of uncertainty in the input parameters simulated by "smearing" the data points:

$$x_i = \overline{x}_i + \sigma_i r_i$$

  - Thickness assumed to have a mean value of 1.6 mm. and an uncertainty of .08 mm.
  - Elastic modulus assumed to have a mean value of 3.3GPa and an uncertainty of 0.33GPa

Computational Model continued

- **Aeroelastic modeling**
  - Smaller model used in order to allow for comparison with conventional Monte Carlo method
  - Total of 4 out-of-plane and 20 in-plane structural modes used in modal solution
  - 100 bound vortex rings used in aerodynamic model
  - Total of 124 degrees of freedom in computational model
    - Typical production aeroelastic model would have between
    - 10^4 and 10^7 degrees of freedom

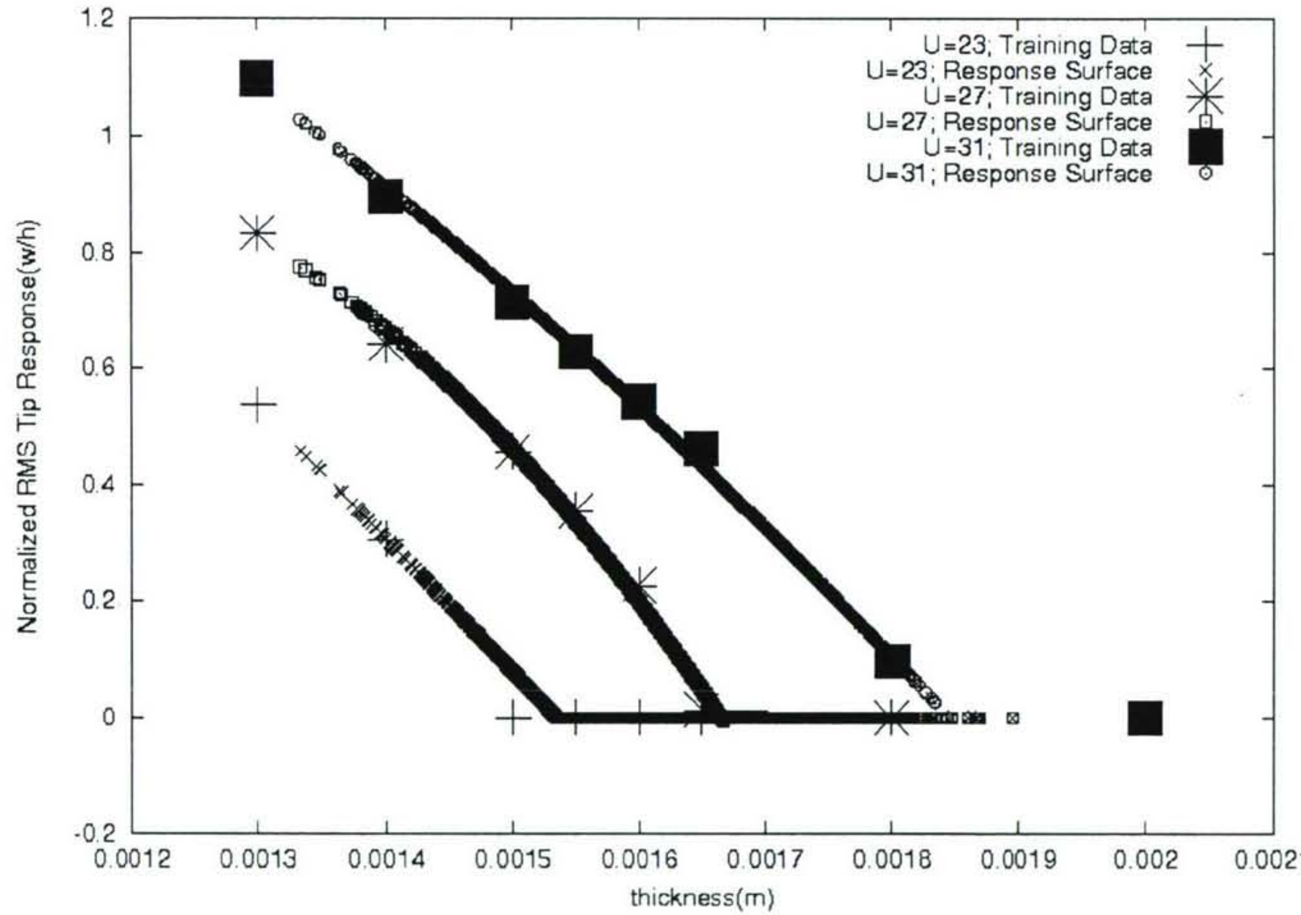
Training the RSM

- **In order to train the response surface model, N values chosen for input parameters**
  - N should be at least as large as number of coefficients in response surface model
- **Currently values chosen by inspection**
  - method such as central composite design (CCD) could be used to determine the N values used
- **Value of N determines how many Monte Carlo simulations need to be run on the governing equations**

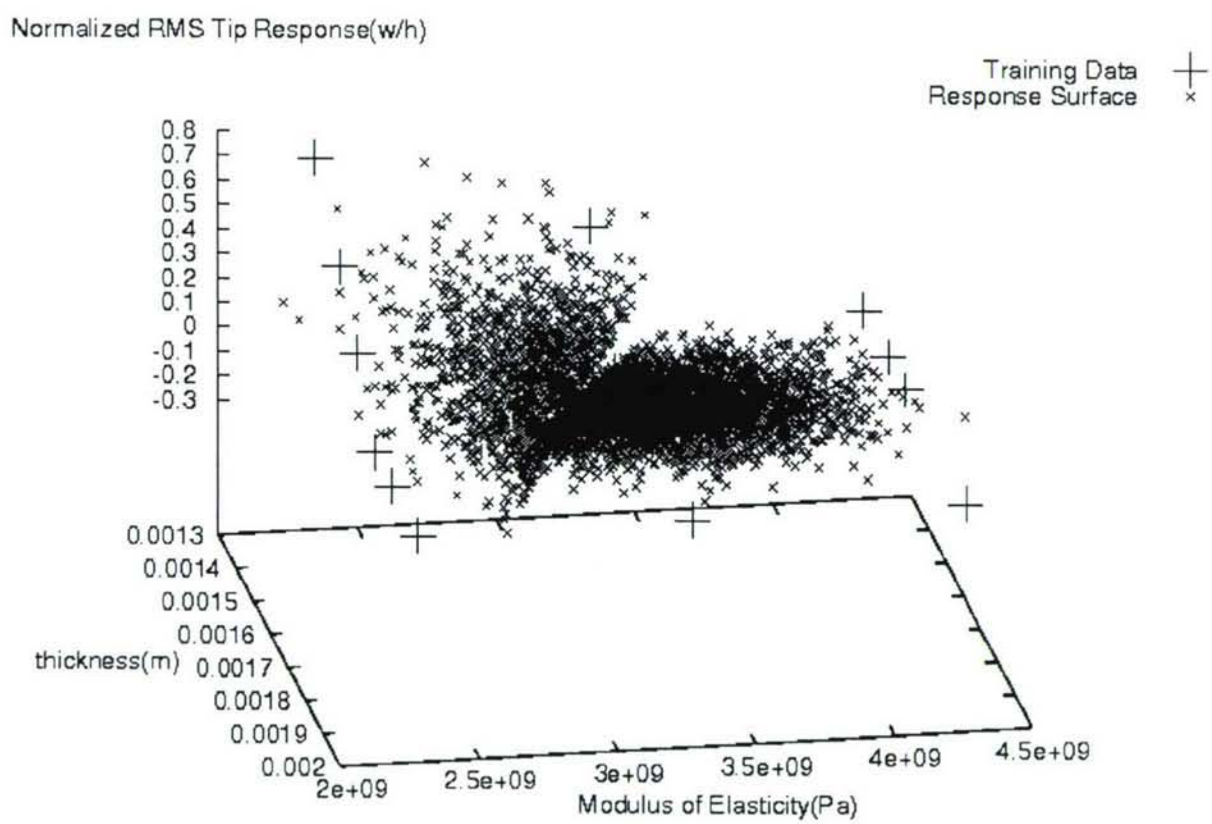
Training the RSM continued…

- **Current work:**
  - One random input parameter: N=8
  - Two random input parameters: N=15
- **Monte Carlo on governing equations:**
  - For each realization of the input parameter(s), the aeroelastic system of equations is integrated for 2 seconds of simulation time
  - Last 25% of time history data used to compute RMS tip deflection

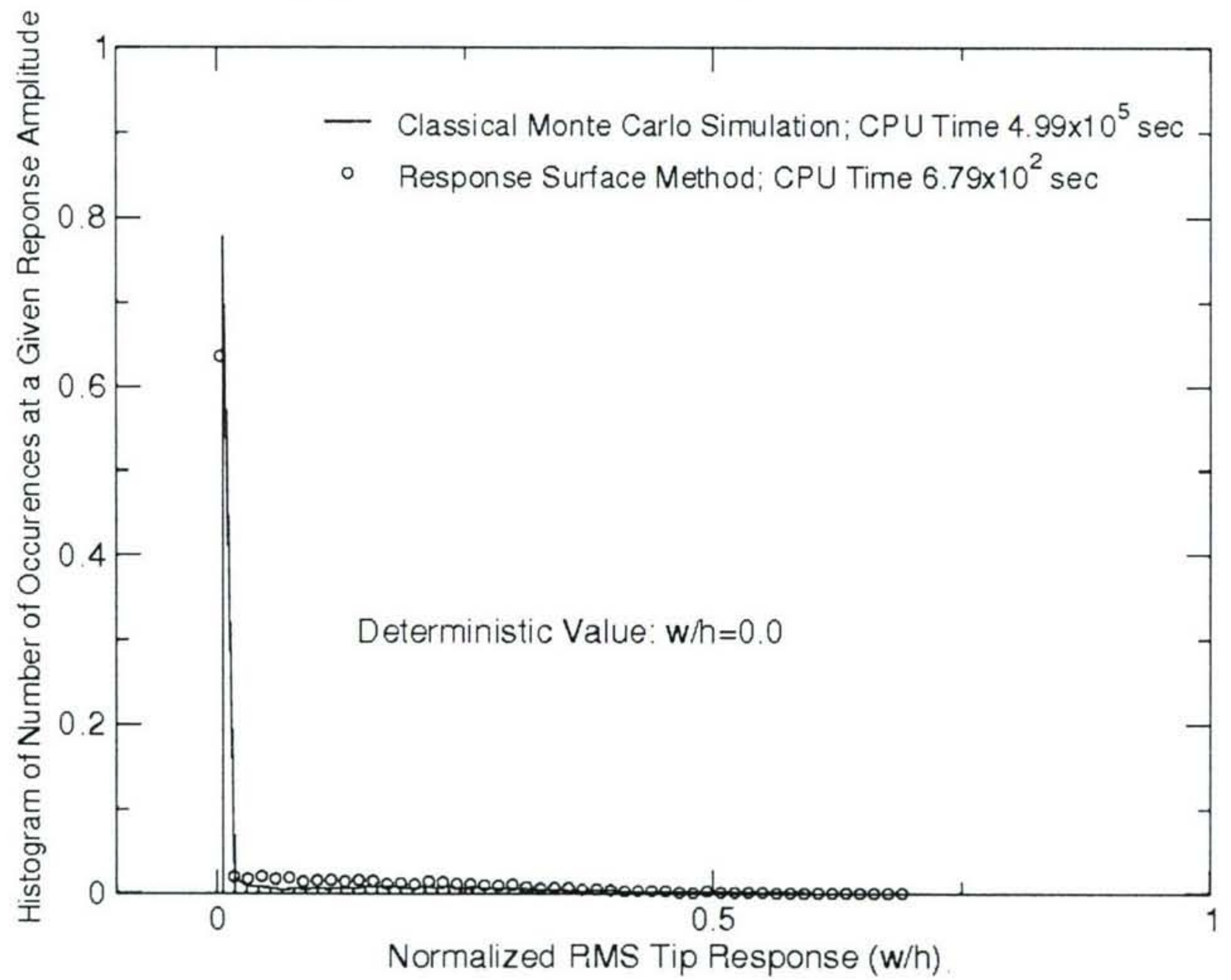
1-D Stochastic Projections



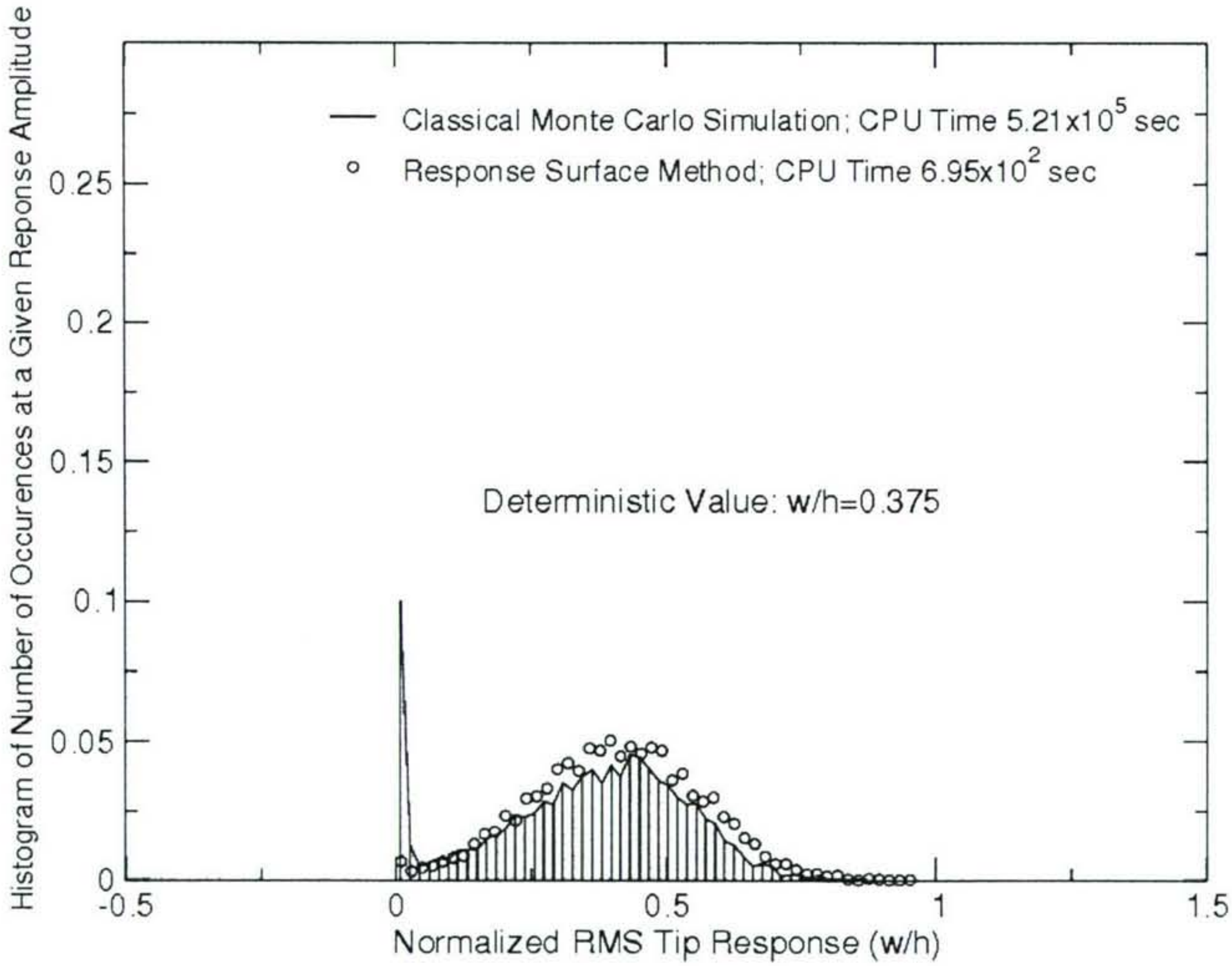
2-D Stochastic Projection



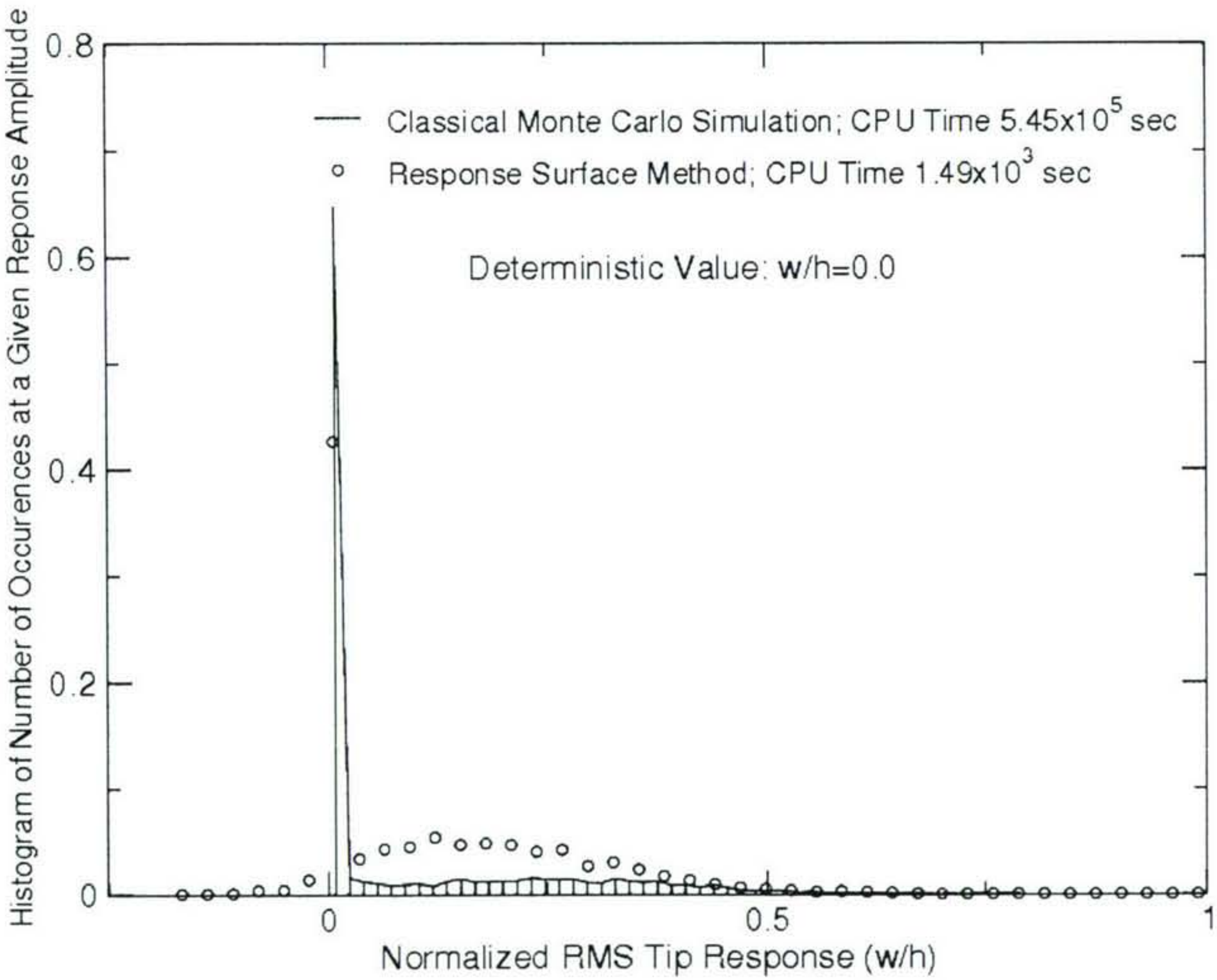
RMS Amplitude Probability 1-D Input Parameter Space Flow Velocity of 25 m/s



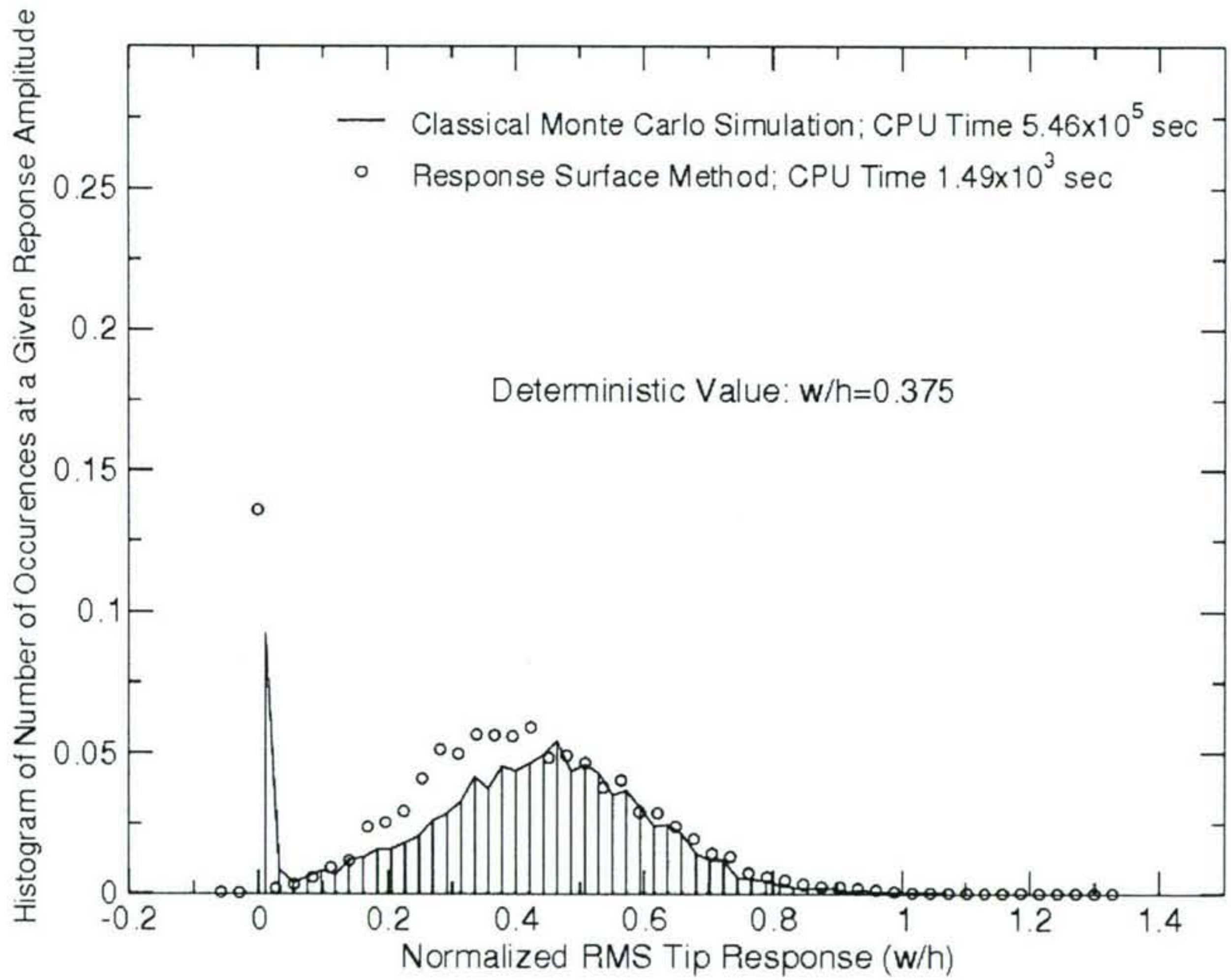
RMS Amplitude Probability 1-D Input Parameter Space Flow Velocity of 29 m/s



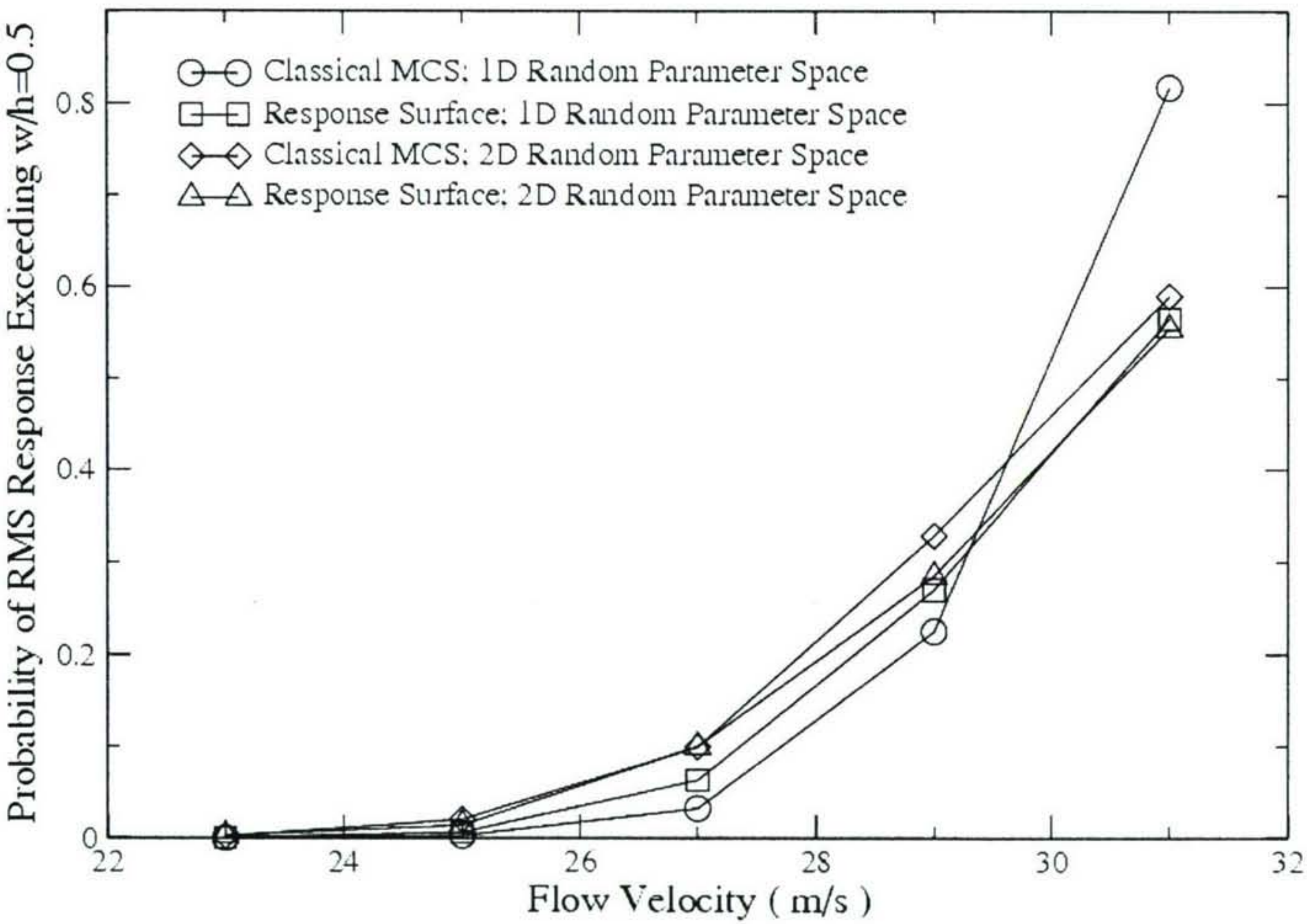
RMS Amplitude Probability 2-D Input Parameter Space Flow Velocity of 25 m/s



RMS Amplitude Probability 2-D Input Parameter Space Flow Velocity of 29 m/s



Probability of “Failure” Plot
Deterministic: U<31 m/s P=0.0, U>31 m/s/ P=100.0



Computational Cost Comparison

	Classical Monte Carlo		RSM	
	1D	2D	1D	2D
# of MCS on governing equations	25000	25000	40	75
# of input variable realizations	25000	25000	25000	25000
Total CPU hours needed to compute failure probability curve	706	1028	0.86	2.40

Summary

- Efficient, non-intrusive method presented for the stochastic analysis of nonlinear aeroelastic phenomena
- Statistical results computed using method compare well to those computed using a Monte Carlo simulation on the governing equations
- Three order of magnitude gain in computational efficiency with current methodology

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH • UNITED STATES AIR FORCE

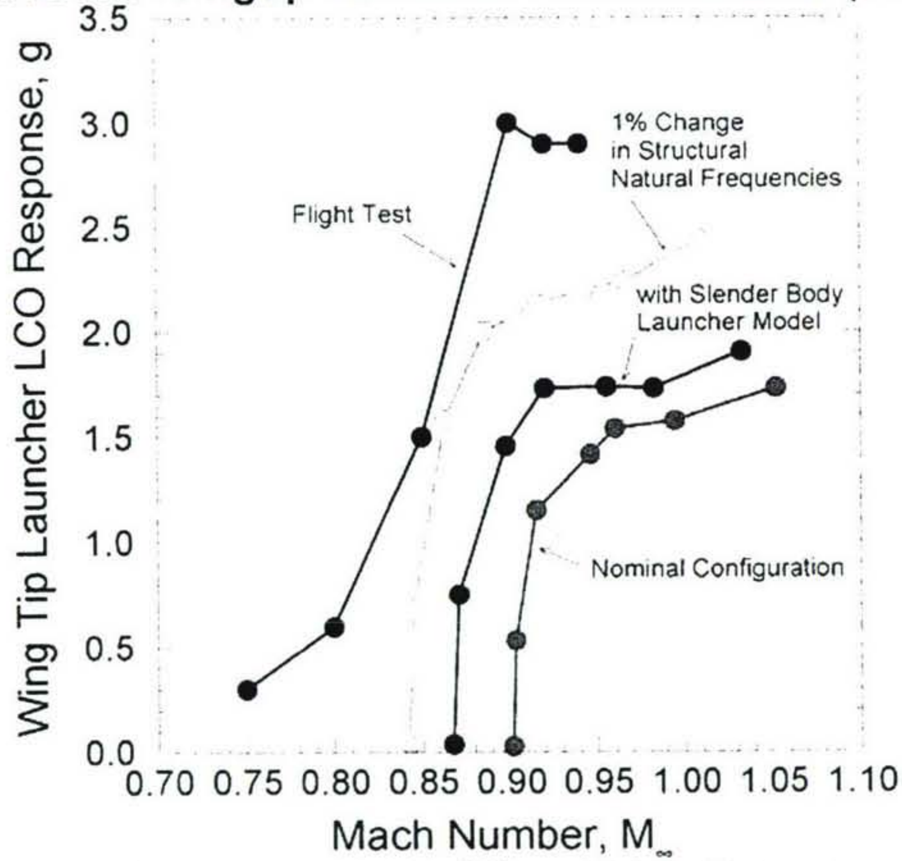
DUKE
EDMUND T. PRATT, JR.
SCHOOL OF ENGINEERING

F-16 SENSITIVITY ANALYSES

LCO Response vs Mach Number: Sensitivity to Uncertainty in Aerodynamic Modeling of Tip Missile and Structural Natural Frequencies

Computed and Experimental LCO Response Trends

F-16 Forward Wingtip Launcher Accelerometer (2000 feet)



F-16 (Block 40) Experimental LCO Aircraft Weapons and Stores Configurations

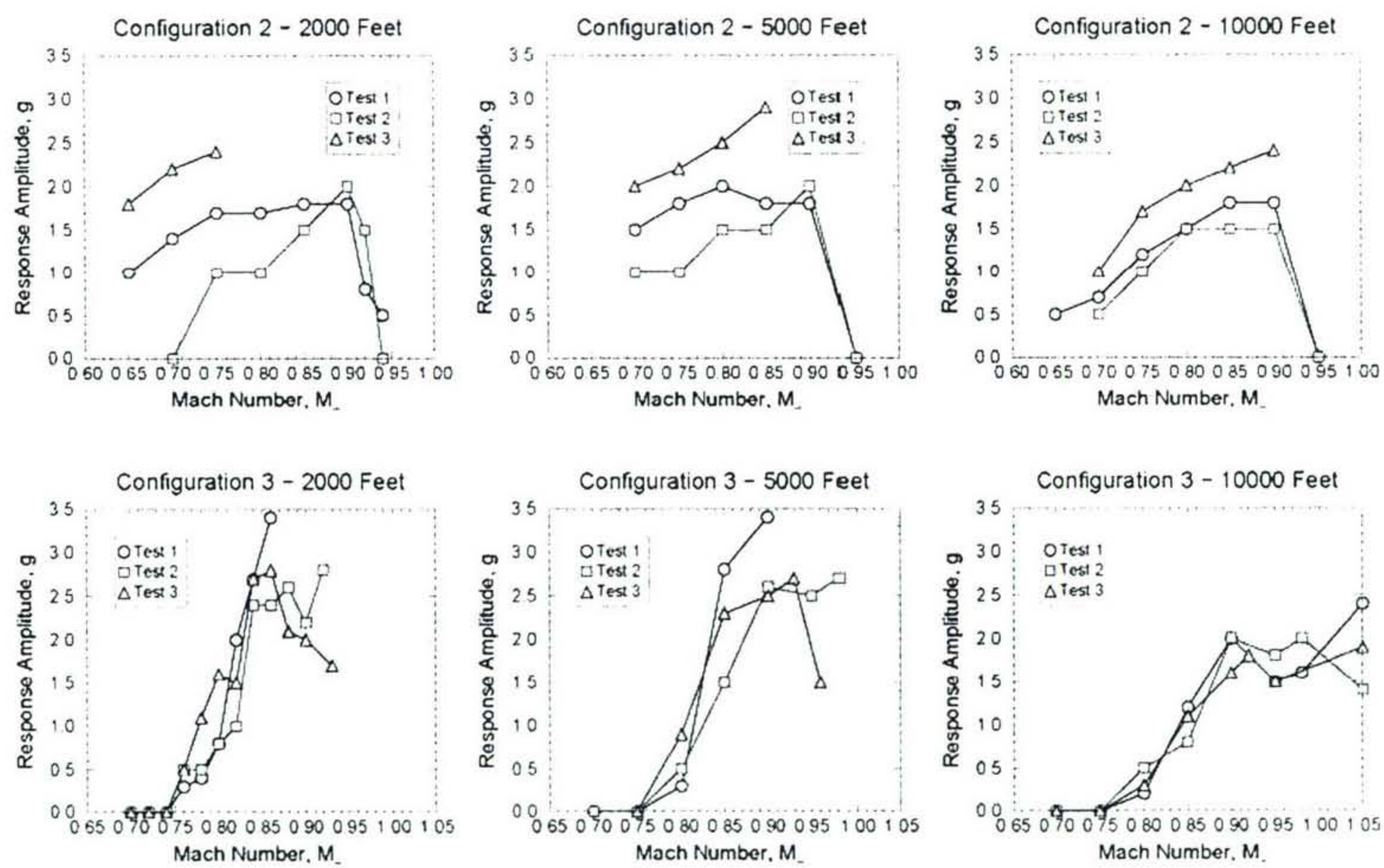
Stn.	Configuration 1	Configuration 2	Configuration 3
1	LAU-129 launcher	AIM-9L missile/LAU-129 launcher	LAU-129 launcher
2	AIM-9P missile/LAU-129 launcher	AIM-9L missile/LAU-129 launcher	AIM-120 missile/LAU-129 launcher
3	Air-to-ground missile	Air-to-ground missile	General purpose bomb
4	Empty 370-gal fuel tank	Half-full 370-gal fuel tank	Quarter-full 370-gal fuel tank
5	Empty station	Empty station	Empty station
6	Empty 370-gal fuel tank	Half-full 370-gal fuel tank	Quarter-full 370-gal fuel tank
7	Air-to-ground missile	Air-to-ground missile	General purpose bomb
8	AIM-9P missile/LAU-129 launcher	AIM-9L missile/LAU-129 launcher	AIM-120 missile/LAU-129 launcher
9	LAU-129 launcher	AIM-9L missile/LAU-129 launcher	LAU-129 launcher

F-16C Configuration Natural Frequencies

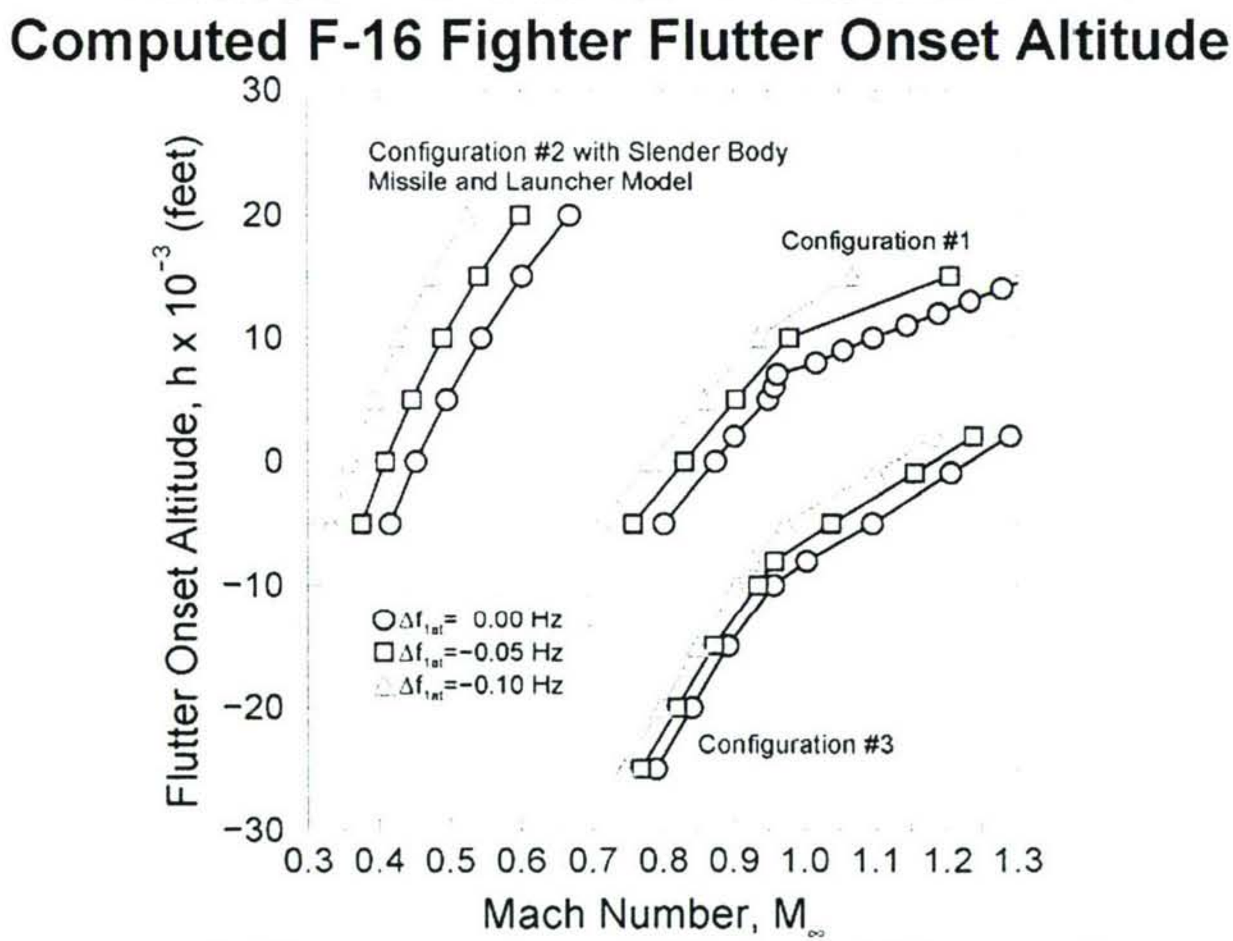
Mode	Configuration 1	Configuration 2	Configuration 3
First Bending (f_{1ab})	8.17 Hz	5.47 Hz	6.50 Hz
First Twisting (f_{1at})	8.67 Hz	5.74 Hz	7.32 Hz
Second Bending (f_{2ab})	10.9 Hz	7.87 Hz	8.37 Hz
Second Twisting (f_{2at})	12.3 Hz	8.01 Hz	8.97 Hz
$f_{1at} - f_{1ab}$	0.504 Hz	0.265 Hz	0.820 Hz

Antisymmetric Modes Via NASTRAN

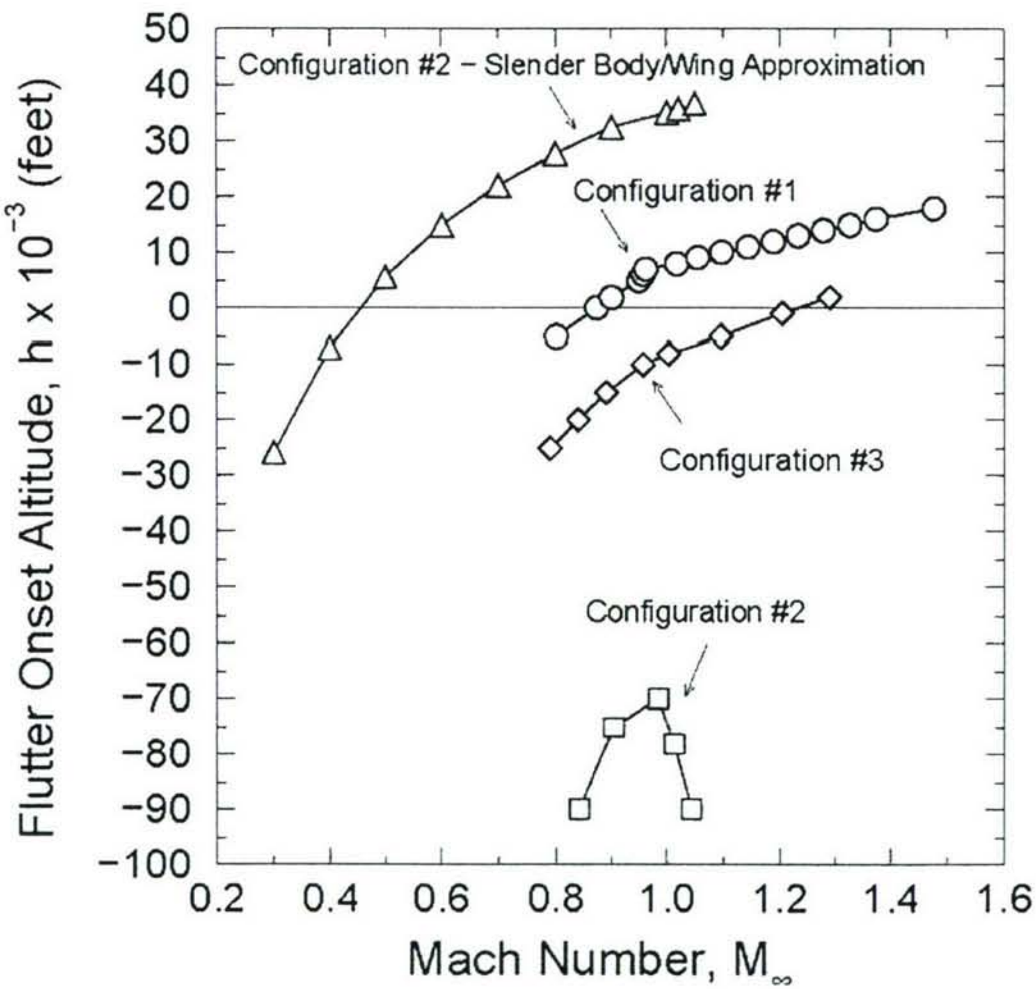
F-16 Flight Test Wingtip LCO Response



STRUCTURAL NATURAL FREQUENCY EFFECT



Flutter Onset Altitude vs Mach Number



CONCLUSIONS

A New Methodology has demonstrated (quantitatively) the Sensitivity of Linear and Nonlinear Aeroelastic Response to

- Uncertainties in Structural Natural Frequencies
- Aerodynamic Modeling of Tip Missiles